NOTES AND CORRESPONDENCE

On the Consistent Scaling of Terms in the Sea-Ice Dynamics Equation

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ABSTRACT

The standard way in which the sea-ice dynamics equation is used in models assumes that the wind stress and ocean drag do not depend on the sea-ice concentration. It is demonstrated that this assumption is inconsistent with the free-drift limit, and how great an effect it has in practice is examined. By examining the momentum balance in the free-drift limit, the authors determine the proper area scaling for the forcing terms, thereby obtaining a more accurate solution, particularly in low-ice-concentration regions.

1. The sea-ice momentum equation

The basic equation for sea-ice momentum is often used in models in the form (e.g., Hibler 1979, hereinafter H79)

$$0 = (\text{rheology}) + \boldsymbol{\tau}_a + \boldsymbol{\tau}_w - \rho Hcf\mathbf{k} \times \mathbf{u}, \qquad (1)$$

where $\mathbf{u} = (u, v) =$ ice velocity, $\rho =$ ice density, $H =$ ice thickness averaged over the ice area, $f =$ Coriolis parameter, $\mathbf{k} =$ vertical unit vector, and $c =$ ice concentration (i.e., fraction). The wind stress $\boldsymbol{\tau}_a$ and the ice–ocean stress $\boldsymbol{\tau}_w$ are typically approximated with a quadratic dependence on wind and ocean current velocities, respectively. Their exact form will not be important for this paper.

Note that H, ice thickness averaged over ice, is the quantity directly related to measurements of thickness (which we are assuming is uniform within the grid cell), whereas Hc, ice thickness averaged over the grid cell, is a computational quantity.

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Equation (1) expresses the momentum balance averaged over the model grid cell, and all its terms are in newtons per meter squared. This form disregards the sea surface tilt term and assumes the net acceleration is negligible (Rothrock 1975). For the sake of simplicity, we consider the case of snow-free ice. We shall be thinking of the rheology term as being of the viscous–plastic or elastic–viscous–plastic (EVP) form, but this assumption will not be important.

2. Statement of the problem

To simplify the equations further, consider the ‘‘free drift’’ case in which the rheology term is neglected (this can be achieved formally by setting the ice strength parameter to zero) and assume all ice has uniform thickness; the remainder of the grid box is open water. Equation (1) then becomes

$$0 = \boldsymbol{\tau}_a + \boldsymbol{\tau}_w - \rho Hcf\mathbf{k} \times \mathbf{u}. \qquad (2)$$

The Coriolis term depends on the ice concentration, but the forcing terms $\boldsymbol{\tau}_a$ and $\boldsymbol{\tau}_w$ do not. As a consequence, the solution of (2) depends on c. For clarity, the appendix displays the solution of (2) explicitly for the case of linear drag.

To see that this situation is a problem, consider a group of ice floes in a region with a given constant ice thickness H and with a concentration of 10%. This ice is moving with a velocity $\mathbf{u}$ given by the balance of wind stress, ocean–ice stress, and Coriolis force [(2)], in a state of free drift. If the ice concentration were only 5% (but the thickness H remained constant) the ice floes should be drifting with the same velocity. However, (2) predicts a different velocity because the factor c appears in the Coriolis term. Because we are considering the case of free drift, in which ice floes do not interact, this should not be the case. Thus (1), with component terms as defined above in the ''usual'' way, is not consistent with the free-drift limit, as it should be.

3. Resolution of the problem

By considering the free-drift limit, this problem is quickly resolved. In free drift, floes do not interact, and the solution should be the same with or without leads: thus the most natural form for the free-drift equations has the wind and water stress, and the mass, averaged *per unit area of sea ice, not per unit area of the grid cell.* To achieve this condition amounts to using the ice thickness averaged over only the ice area, rather than the gridbox mean ice thickness, in the Coriolis term.

Equivalently, the forcing terms $\boldsymbol{\tau}_a$ and $\boldsymbol{\tau}_w$ in (1) and (2) may be multiplied by the ice concentration. In this case, the equation is interpreted still as averaged over the gridbox area, but the proportion of the grid box that is ice free does not contribute to the wind or water stress terms. In effect, this means (1) has been applied separately to the ice-covered and ice-free areas; in the latter, because the ice thickness is zero, the rheology and Coriolis terms are zero and we simply have $\boldsymbol{\tau}_a + \boldsymbol{\tau}_w = 0$.

For clarity, we now give the correct form of the equation and terms, which is

$$0 = (\text{rheology}) + c(\boldsymbol{\tau}_a + \boldsymbol{\tau}_w) - \rho H c f \mathbf{k} \times \mathbf{u}. \quad (3)$$

Equation (3) is the same as (1), but the terms $\boldsymbol{\tau}_a$ and $\boldsymbol{\tau}_w$ are scaled by a factor of c. For convenience we also give the corrected equation in the case of free drift:

$$0 = c(\boldsymbol{\tau}_a + \boldsymbol{\tau}_w) - \rho H c f \mathbf{k} \times \mathbf{u}. \quad (4)$$

This form lends itself to implementation in coupled ice–ocean–atmosphere models, in which the atmospheric stress is split into the stress over leads $[(1 - c)\boldsymbol{\tau}_a]$, which is passed straight into the ocean, and the stress over the ice ($c\boldsymbol{\tau}_a$), which goes through the sea-ice model. In a similar way, the ice–ocean stress that is passed from the ice model into the ocean is only applicable over the area covered by ice.

In the case we have been considering of thick ice in free drift, the correct formulation for the ice mass is simple to see. The situation becomes less obvious when we consider multicategory ice. When the modeled sea ice is considered to consist of just two categories, ''thick ice'' and ''thin ice,'' rather than ''thick ice'' and ''open water,'' the thin ice is still generally assumed to have no strength, and so it must experience zero net stress ($\boldsymbol{\tau}_a + \boldsymbol{\tau}_w = 0$) and the situation is unaltered. Floes of different nonzero thickness in free drift should have different velocities, however; a multicategory free-drift code could solve the momentum balance (3) separately for each thickness. In practice, however, multicategory ice is only considered when rheology is also taken into account.

When the rheology term is included, it is necessary to use the area-averaged form of the momentum balance, because the rheology term intrinsically depends on a continuum viewpoint with no distinction between ice and leads. A single velocity is used for all thickness categories because there is a single strain-rate tensor for the ice continuum. This approach is obviously an approximation, but it seems unavoidable with current formulations of rheology. We maintain that in the multicategory case, (3) is still correct, because the ice-free area (or ''thin ice'') has no strength and hence does not affect the rheology term in (3).

Our approach is equivalent to applying the wind stress over the leads to the ocean momentum budget, decoupling the water in the leads from the ice momentum budget. In reality, it is likely that there is substantial lateral drag on the ice floes, tending to make the ice and the water within the leads move with similar velocities (Gray and Morland 1994). This fact could be an argument for including the leads in the momentum budget with $\boldsymbol{\tau}_a$ and $\boldsymbol{\tau}_w$ not multiplied by c. However, in that case the appropriate mass in (1) is not the ice mass, but the mass of the combined slab of ice and water, assumed to move as a rigid body, with mass per unit area ρH because, by isostasy, the water in the leads from the base of the floe to the surface has the same mass per unit area as the sea ice. Hence the combined momentum balance (in the free-drift case) takes the form

$$0 = (\boldsymbol{\tau}_a + \boldsymbol{\tau}_w) - \rho H f \mathbf{k} \times \mathbf{u}. \quad (5)$$

In multiplying through by c, (5) is converted to (4), and so our velocity solution is unchanged. The solution given by (2) for this case is still different, though, because it neglects the mass of the water in the leads. This discussion of the wind stress over the leads is also a simplification for many reasons: for instance, the ocean drag on the leads water will not have the same form as on the ice, and it is only the areas of leads reasonably near to ice floes that will be dragged along with them. As with the multicategory ice, the use of a single momentum equation for a continuum with a single velocity is of course the basic inadequacy.

The problem regarding the treatment of stresses appears to originate in the paper of H79 and follows through to many, but not all, papers following this work (e.g., Hunke and Dukowicz 1997). It also appears in some papers (e.g., Overland and Pease 1988) that do

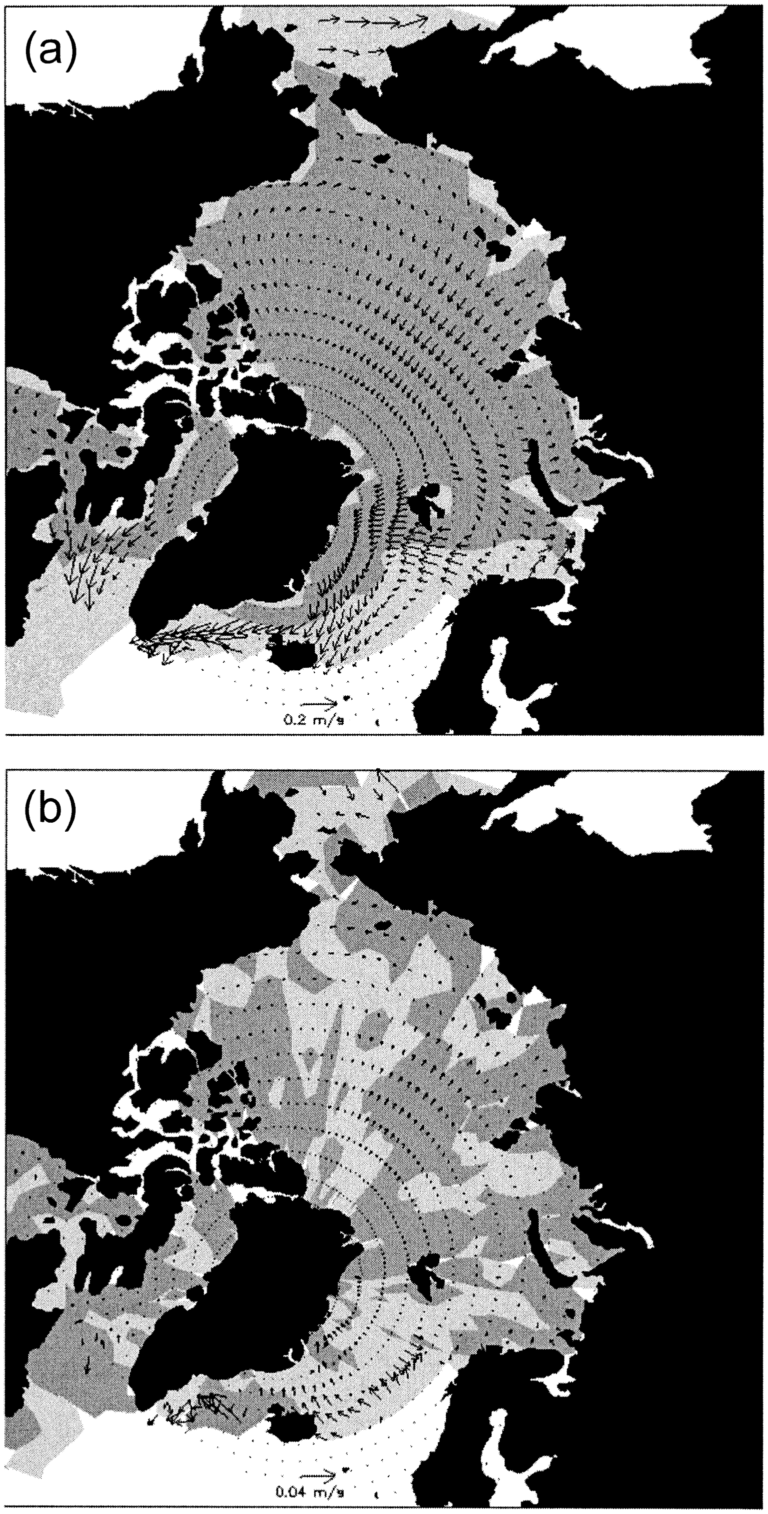

FIG. 1. Arctic ice area fraction and velocity (m s^{-1}) for Jan, year 30, from the POP/CICE model. (a) Control: light (dark) shading indicates ice area of 0–85% (85%–100%). (b) Test − control: shading indicates area difference (light, negative; dark, positive). Note the change in scale arrow between (a) and (b).

not explicitly follow H79, however. In these cases, $\boldsymbol{\tau}_a$ and $\boldsymbol{\tau}_w$ were recognized as approximate, cell-average quantities, without the realization that an improved approximation, consistent in the free-drift limit, results from applying $\boldsymbol{\tau}_a + \boldsymbol{\tau}_w = 0$ over the open-water area. In most other cases, it is unclear from papers whether the problem is present, because the ice thickness h is often quoted as the ''average ice thickness'' and the crucial distinction between ''average over the grid box area'' and ''average over the ice area'' cannot be clearly made. A few papers (e.g., Hakkinen 1987; Haapala 2000) can be seen to be correct. Only one paper of which we are aware (Gray and Morland 1994, p. 267) shows awareness of the problem; however, the analysis is buried deeply within the paper and has not been picked up by the community.

4. Practical effects

The correction proposed here amounts to multiplying the drag terms of the ice momentum equation by the ice concentration. In practice, the ice concentration is often 90% or higher within the pack, and thus for large areas the change to the equations would be small.

We perform two anomaly integrations to test this (the control integration uses the corrected form of the equations; the anomaly uses the uncorrected form). The first uses a coupled atmosphere–ocean–ice GCM [Hadley Centre Coupled Model (HadCM3; Gordon et al. 2000) with EVP sea-ice dynamics]. This approach has the disadvantage of nonrepeatability: stochastic interannual variation within the coupled system means that the difference between individual years may be due to this variation rather than a reflection of the change in the equations. To minimize this effect, we use an average of 5 yr. However, it has the benefit of allowing atmospheric feedback, to test the possibility that relatively small change could lead by feedback to larger effects. The results from this run are not shown. Changes are small and cannot be distinguished from interannual variability.

The second test uses an ocean–ice GCM with imposed atmospheric forcing [Parallel Ocean Program/Los Alamos Sea Ice Model (POP/CICE); Hunke and Lipscomb 2001; Smith and Gent 2002]. This setup has the benefit that differences between control and anomaly at year 30 represent the results of the change in the equations alone; there is no atmospheric feedback.

The results from the POP/CICE test are shown in Fig. 1, for January of year 30. Differences in the ice area are minor except near the ice edge, where the concentration is less than about 90%; the magnitude of the differences lies between −1% and 1% nearly everywhere. The biggest difference for ice velocity appears to be direction. Reduced wind stress would make the ice drift more slowly, but reduced ocean drag compensates for that somewhat; the Coriolis term in the test run is more important relative to the wind and ocean stresses than in the control run, resulting in turning of the velocity vectors.

5. Conclusions

To make the sea-ice dynamics equation consistent with the free-drift limit, the wind stress and ocean drag terms should be multiplied by the sea-ice concentration. This correction to a model is small and easily implemented. The effects in practice are not large, but it is preferable to use a model that treats the low-concentration limit of free drift correctly as well as the high-concentration situations in which the rheology comes into play and the correction is relatively less important.

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APPENDIX

The Solution of (2) in the Case of Linear Drag

If we assume for simplicity that $\mathbf{u}_w$ is zero, and we assume a linear drag form for $\boldsymbol{\tau}_w$, that is,

$$\boldsymbol{\tau}_w = -d\mathbf{u},$$

with d a constant, then (2) becomes

$$0 = \boldsymbol{\tau}_a - d\mathbf{u} - \rho Hcf\mathbf{k} \times \mathbf{u},$$

which can be rearranged as

$$d\mathbf{u} + \rho Hcf\mathbf{k} \times \mathbf{u} = \boldsymbol{\tau}_a.$$

Hence, taking the cross product with $\rho Hcf\mathbf{k}$,

$$\mathbf{u} = (d - \rho Hcf\mathbf{k}\, \times)\boldsymbol{\tau}_a/[d^2 + (\rho Hcf)^2]. \tag{A1}$$

Since $\mathbf{k} \times \boldsymbol{\tau}_a$ is perpendicular to $\boldsymbol{\tau}_a$,

$$|\mathbf{u}| = |\boldsymbol{\tau}_a|/[d^2 + (\rho Hcf)^2]^{1/2}, \tag{A2}$$

and the opening angle clockwise from $\boldsymbol{\tau}_a$ to $\mathbf{u}$ is

$$\mathrm{atan}[-(\rho Hcf)/d]. \tag{A3}$$

Equations (A1)–(A3) are seen to depend on c, which they should not. Because our correction multiplies both $\boldsymbol{\tau}_a$ and d by c but does not change (ρHcf), it is clear that both the magnitude of $\mathbf{u}$ and the opening angle are affected by our correction: $|\mathbf{u}|$ is decreased and the opening angle is increased. Equations (A1)–(A3) are the solution for the incorrect form (2); the solution corresponding to (A3) for the corrected (4) multiplies both $\boldsymbol{\tau}_a$ and d by c and hence is

$$\mathbf{u} = (d - \rho Hf\mathbf{k}\, \times)\boldsymbol{\tau}_a/[d^2 + (\rho Hf)^2]^{1/2}.$$

This solution (correctly) does not depend on the ice concentration c.

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